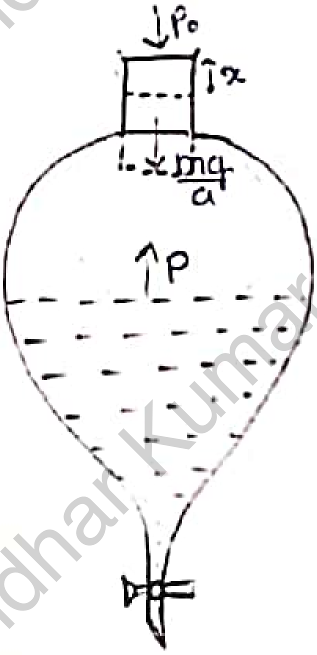
Helmholtz Resonator:

It works on the principle of vibration of air column. Helmholtz Resonator is closed vessel having two openings, the cylindrical neck at the top & the narrow pipe at the bottom, having stopper. Through which volume of air column can be varied.

When the vibrating tuning fork is held at the mouth of the resonator it sounds loudly. when the natural frequency of air column is equals to frequency of tuning fork. This process is called as Resonance.

Let us consider Helmholtz Resonator having the neck of cross-sectional area "a" and let the mass of the air contained in the neck is "m" and rest of the volume of air be "V" and let pressure inside the resonator is p "P" and pressure outside the resonator is "P_0" and the pressure because of air in the neck is "$\frac{mg}{a}$".

At equilibrium, $P = P_0 + \frac{mg}{a}$ —— ①



When the air in the flask is in resonance with particular frequency then air in the neck will move up and down. It acts like a piston.

Let at any instant the air column in the neck moves downwards by the distance "x"

$\therefore V_{new} = V - ax$

If the process is adiabatic then we can write,

Let P_1 be the new pressure.

(15) $P_1(V-ax)^{\gamma} = P(V)^{\gamma}$ —— (2) $\boxed{\gamma = \frac{C_P}{C_V}}$

$$P_1 = \frac{PV^{\gamma}}{(V-ax)^{\gamma}}$$

$$P_1 = P\left(\frac{V}{V-ax}\right)^{\gamma}$$

$$P_1 = P\left[1 + \frac{ax}{V-ax}\right]^{\gamma}$$

By using Binomial expansion, we get

$$P_1 = P\left[1 + \frac{\gamma\, ax}{V-ax}\right]$$

$$P_1 = P + \frac{Pax\gamma}{V-ax}$$

$$P_1 - P = \frac{P\gamma ax}{V-ax} \quad \text{—— (3)}$$

$\left\{ (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \ldots\ldots \right.$ x is very very small then. $\boxed{(1+x)^n = 1+nx}$

The net downward force "F" on the air in the resonator is.

$$F = (P - P_1)\, a$$

$$\frac{F}{a} = P - P_1$$

$\left\{ \text{Pressure} = \frac{F}{a} \right\}$

Now substituting the value of $P - P_1$ from eqn (3)

$$\frac{F}{a} = \frac{-P\gamma ax}{V-ax}$$

$$F = \frac{-P\gamma a^2 x}{V-ax}$$

As ax is very small as compared to V So we can neglect ax in the denominator of above eqn.

$$F = -\frac{P\gamma a^2 x}{V}$$

$$\text{Mass} \times \text{Acceleration} = \frac{-P\gamma a^2 x}{V}$$

$$m \times \text{acceleration} = \frac{-P\gamma a^2 x}{V}$$

(10) acceleration $= \dfrac{-P\gamma a^2 x}{Vm}$

acceleration $= \dfrac{-P\gamma a^2}{Vm} x$

Then formula of time

$$T = 2\pi\sqrt{\frac{d}{a}}$$

The above eqⁿ represents SHM.

$$\frac{\text{acceleration}}{x} = \frac{-P\gamma a^2}{Vm}$$

(−) sign indicates the pressure moves downwards

Now, $T = 2\pi\sqrt{\dfrac{d}{a}}$

$$T = 2\pi\sqrt{\frac{Vm}{P\gamma a^2}}$$

frequency, $f = \dfrac{1}{T}$

$$f = \frac{1}{2\pi}\sqrt{\frac{P\gamma a^2}{Vm}} \quad \text{——— (4)}$$

The velocity of sound in the air is given by. $v = \sqrt{\dfrac{K_s}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}}$.

where. $K_s \Rightarrow$ Coefficient of stiffness

$K_s = \gamma P$

where, P is pressure

γ is ratio of specific heat at constant pressure to const temp.

& ρ is density of air

$$v^2 = \frac{\gamma P}{\rho}$$

$$\gamma P = v^2 \rho$$

Substitute the value of γP in eqⁿ (4)

$$f = \frac{1}{2\pi}\sqrt{\frac{v^2 \rho a^2}{Vm}} \quad \text{——— (5)}$$

(17) W.K.T. $\rho = \cancel{\frac{m}{al}}\ \frac{m}{al}$

$$\therefore m = \rho a l$$

Substituting the value of m in (5)

$$f = \frac{1}{2\pi}\sqrt{\frac{v^2 \gamma a^2}{V \gamma a l}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{v^2 a}{V l}} = \frac{v}{2\pi}\sqrt{\frac{a}{V l}}$$

$$\boxed{f \propto \frac{1}{\sqrt{V}}} \quad \text{------} \quad (6)$$

By knowing the values of length & ~~area~~ cross-sectional area of the neck & volume of resonator V. we can find out unknown frequency.

Eq^n (6) shows that frequency is inversely proportional to square root of volume V.